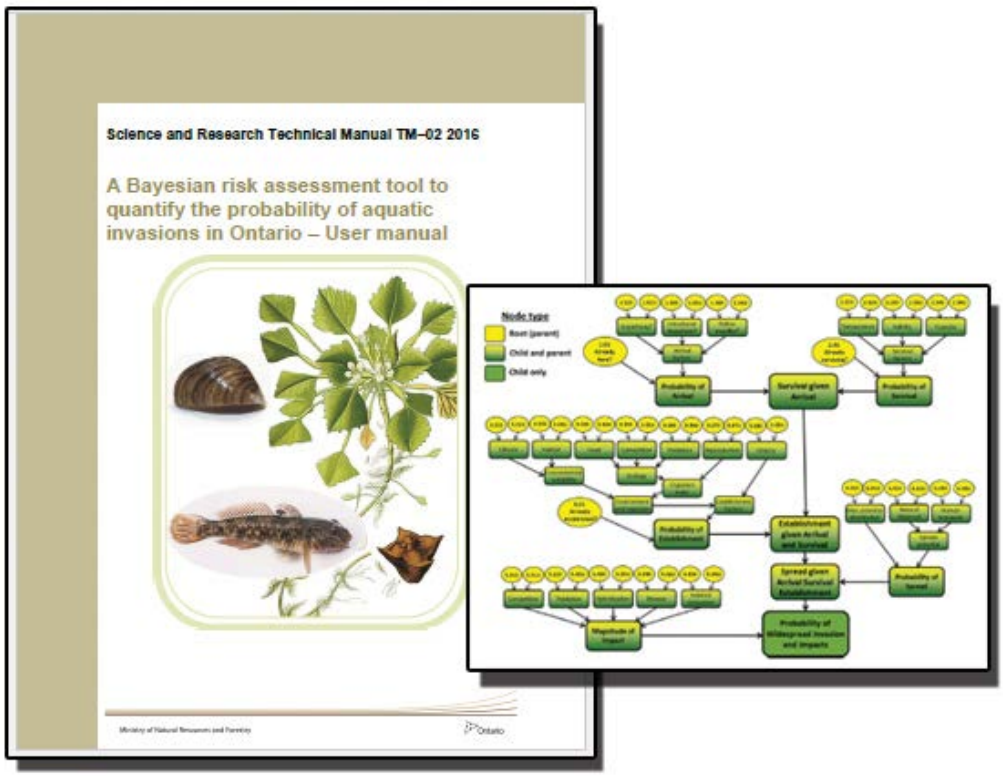
Science and Research Information Report IR-09

A Bayesian risk assessment tool to quantify the probability of aquatic invasions in Ontario – Background documentation



Ministry of Natural Resources and Forestry

Ontario

Science and Research Information Report IR-09

A Bayesian risk assessment tool to quantify the probability of aquatic invasions in Ontario – Background documentation

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Aquatic Research and Monitoring Section

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Science and Research Branch

Ministry of Natural Resources and Forestry

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Abstract

A quantitative, detailed-level risk assessment approach for evaluating the likelihood of invasion and impacts of AIS in Ontario. The framework follows a questionnaire-style format complete with detailed guidance and examples to comprehensively address all stages in the invasion process: arrival, survival, establishment, spread, and impacts. Answers follow a standardized rating system, require selection of an associated uncertainty level, and are justified through detailed documentation of supporting scientific evidence. A probabilistic modelling approach (i.e., Bayesian risk assessment tool) is applied to integrate answers and uncertainty ratings, combine the questions for all invasion stages, and obtain an overall estimate of risk (presented as probability distribution for the likelihood of invasion and impacts). This approach increases the likelihood that risk assessments are objective, transparent, reproducible, and easily updatable in light of new information.

Résumé

Outil bayésien d’évaluation des risques pour quantifier la probabilité d’invasions aquatiques en Ontario – document d’information.

Méthode quantitative détaillée d’évaluation des risques d’invasions aquatiques et de leurs effets en Ontario. Le cadre suit un document qui prend la forme d’un questionnaire comprenant des directives détaillées et des exemples qui touchent à toutes les étapes du processus d’invasion : arrivée, survie, établissement, propagation et effets. Les réponses s’appuient sur un système de cotation normalisé, demandent la sélection d’un niveau d’incertitude associé et sont justifiées par une documentation détaillée des données scientifiques à l’appui. Une méthode de modélisation probabiliste (c.-à-d. l’outil bayésien d’évaluation des risques) est appliquée pour intégrer les réponses et les coefficients d’incertitude, combiner les questions concernant toutes les étapes d’invasion et obtenir une estimation d’ensemble des risques (présentée comme une distribution des probabilités d’invasion et de dommages). Cette méthode augmente les possibilités que les évaluations des risques soient objectives, transparentes, reproductibles et facilement mises à jour à la lumière de nouveaux renseignements.

Acknowledgements

We thank Mike Hoff (Regional Aquatic Invasive Species Coordinator, United States Fish and Wildlife Service), Cecilia Weibert (Aquatic Invasive Species Risk Assessment Analyst, Michigan Department of Agriculture and Rural Development), Tim Johnson and Shannon Fera (Aquatic Research and Monitoring Section, Science and Research Branch, Ministry of Natural Resources and Forestry [MNRF]), and Jeff Brinsmead and Francine MacDonald (Natural Heritage Section, Natural Resources Conservation Policy Branch, MNRF) for reviewing the risk assessment tool and contributing their valued expertise through comments and feedback.

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Contents

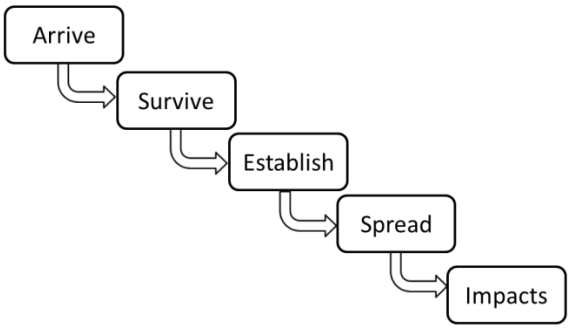
Overview

This document describes the methodology employed to construct the Bayesian risk assessment tool to quantify the probability of aquatic invasions in Ontario. For full transparency, detailed background documentation of the underlying structure and mathematical foundations of the model are presented along with relevant ecological and/or statistical rationale to support decisions or assumptions made during model development. Also presented are model testing results generated by employing the model for various aquatic taxa ranging in their presumed invasion potential for Ontario.

A corresponding user manual (Nienhuis and Haxton 2016) provides detailed instructions for completing the risk assessment and inputting data into the Bayesian risk assessment tool.

Conceptualizing the model for invasion risk

The logical sequence of events in the invasion process follows a hierarchical structure that can be described by a simple conceptual model:

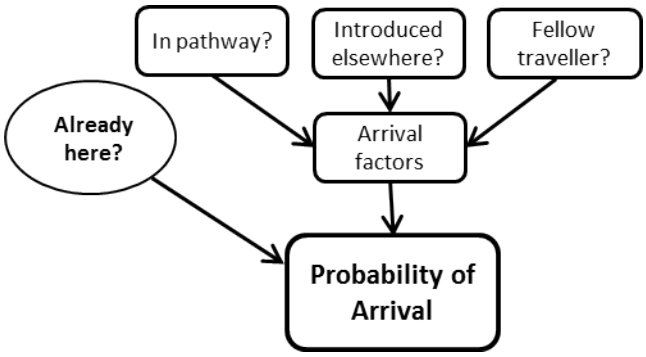


Factors that influence an organism's ability to pass successively from one stage to the next were identified by conducting a review of the relevant literature and consulting existing risk assessment schemes/tools (see Appendix 1 for full list of works consulted).

While essential to the overall process of invasion, the arrival and survival stages are not always explicitly addressed or evaluated in risk assessment schemes. Elsewhere, these are assessed for singular vectors or pathways only (or for particular geographical regions) using purpose-built mathematical models. Here these initial stages are independently incorporated and assessed, but in a manner broad enough to serve as a screening level assessment if necessary.

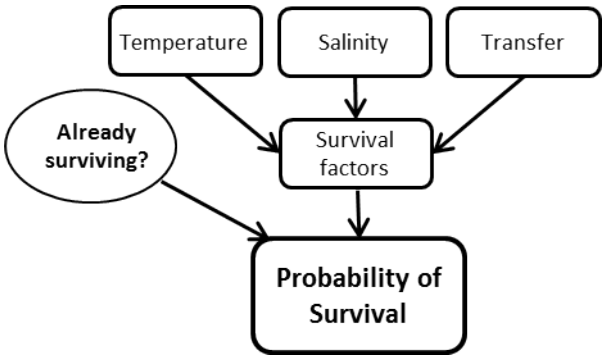
Arrival

Probability of arrival is evaluated by considering whether the species is: 1) already in Ontario; b) in an existing or proposed pathway for arrival; c) known to have been introduced elsewhere through an existing or proposed pathway; or d) a potential fellow traveller or contaminant of goods being transported into Ontario. These factors influence probability of arrival in the following manner:



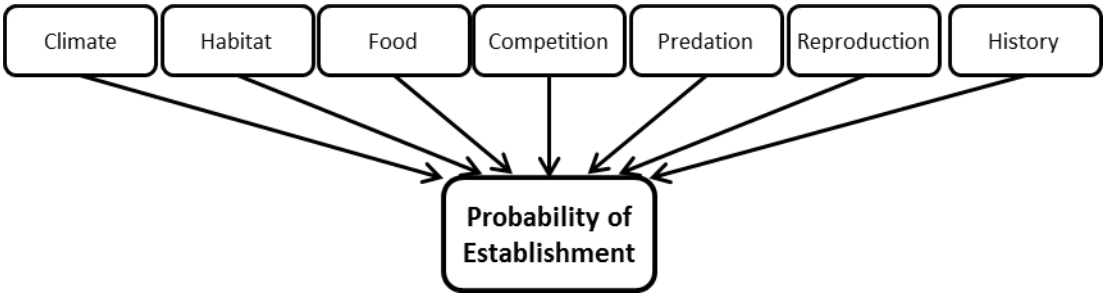
Survival

Factors that would influence a species' propensity for survival are specific to aquatic species within freshwater habitats in Ontario. These include: 1) the ability of any life stage to tolerate temperatures less than 5.5°C (i.e., the general temperature criterion for year-round survival of aquatic organisms in the Great Lakes (Kolar and Lodge 2002; Rixon et al. 2005); 2) the ability to survive in freshwater or brackish habitats (i.e., ruling out strictly marine species (Mandrak et al. 2013); and 3) the ability to survive transfer from a pathway to the natural environment. The model also accounts for cases where a species is already known to be able to survive year-round in the natural environment in Ontario. The contribution of these factors to the overall probability of survival is conceptualized as follows:

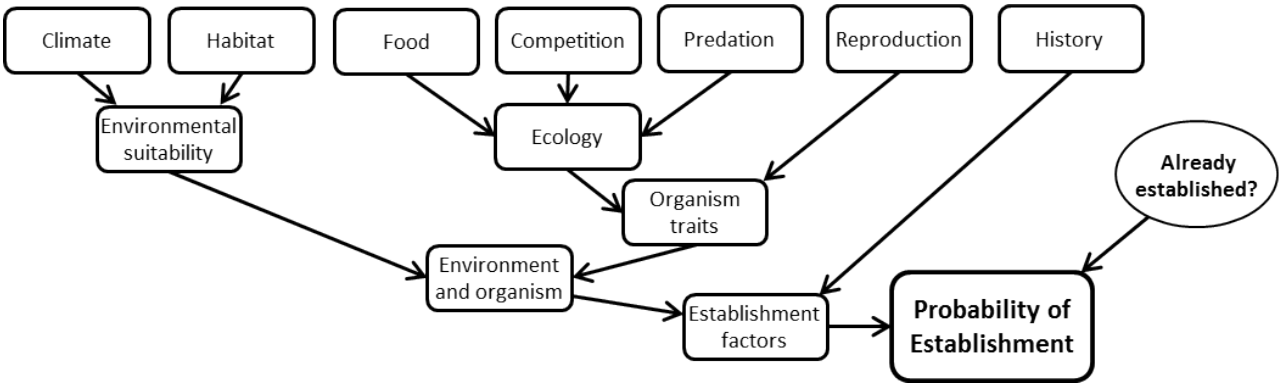


Establishment

Establishment potential is influenced by the taxonomic, physiological, and/or ecological traits of the species as well the climatic, biogeographic, habitat, and/or ecological suitability of the risk assessment area. In addition, one of the strongest predictors of establishment (or invasion) success is a history of establishment or invasion elsewhere. Most existing risk assessment schemes include an evaluation of some or all of these factors. A list of questions relating to establishment was compiled from consulted risk assessment schemes/ questionnaires and grouped according to broad, generalizable categories. The categories chosen were comprehensive yet parsimonious and represent the factors included in the model for establishment probability. These are:

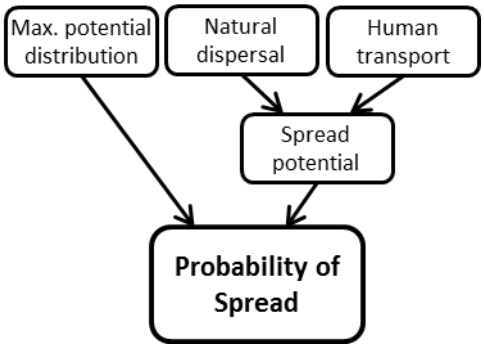


Related factors were grouped hierarchically for ease of modelling (i.e., to reduce the number of states in conditional probability tables for the establishment node — see below for definition) and to conceptualize the relationships among them. The model also accounts for the case in which a species is documented to have already established at least one self-sustaining population in the natural environment in Ontario.



Spread

Factors relating to probability of spread were identified following a similar process used for establishment: by first compiling spread-related questions from existing protocols and then grouping these into the smallest reasonable number of categories. Broadly, there are two discrete mechanisms through which an organism can spread: 1) via natural dispersal; and 2) via human-facilitated movement or dispersal. Together, these define the spread potential of the organism or species. The limit to spread (i.e., the maximum distribution within the risk assessment area) is further influenced by the extent of suitable habitats therein. The overall probability of spread is conceptualized in the following manner:



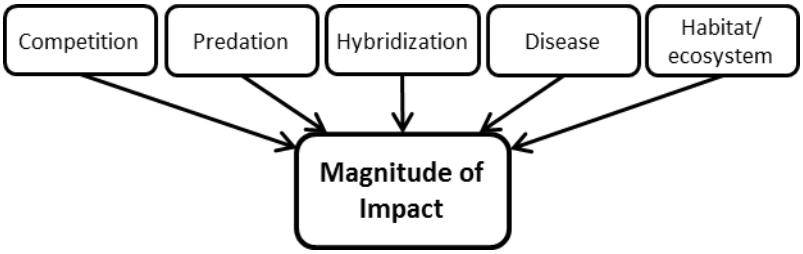
Impacts

An extensive literature review was conducted to identify factors or species traits that influence the likelihood and/or magnitude of ecological impacts. However, the potential for impacts is often highly context dependent (i.e., is strongly influenced by the identity and ecology of both the non-native species and the recipient community or ecosystem), and species traits alone are not always appropriate or consistent predictors of impact (Hayes and Barry 2008).

“Recognizing that impacts vary greatly among species and among recipient systems…a critical need for invasion biology is the capacity to evaluate, compare, and predict the magnitudes of the impacts of different alien species, in order to determine and prioritise appropriate actions where necessary”(Blackburn et al. 2014). To this end, there has been an emerging effort to generalize or standardize the classification of non-native species in terms of the magnitude of their impacts (Nentwig et al. 2009; Blackburn et al. 2014; Kumschick et al. 2012; Kumschick et al. 2015, van der Veer and Nentwig 2015).

A modified version of the generic impact scoring system described by the authors above was used to assess the potential magnitude of impact of introduced species in Ontario because it represents “a consistent procedure for translating the broad range of impact

types and measures into ranked levels of environmental impact" (Blackburn et al. 2014). Overall magnitude of impact is estimated based on evaluation of five broad potential mechanisms of impact:



Overall conceptual model

Having identified a comprehensive set of influential factors for each invasion stage, these were hierarchically linked together according to the full sequence of events in the invasion process. The full conceptual model is shown in Figure 1.

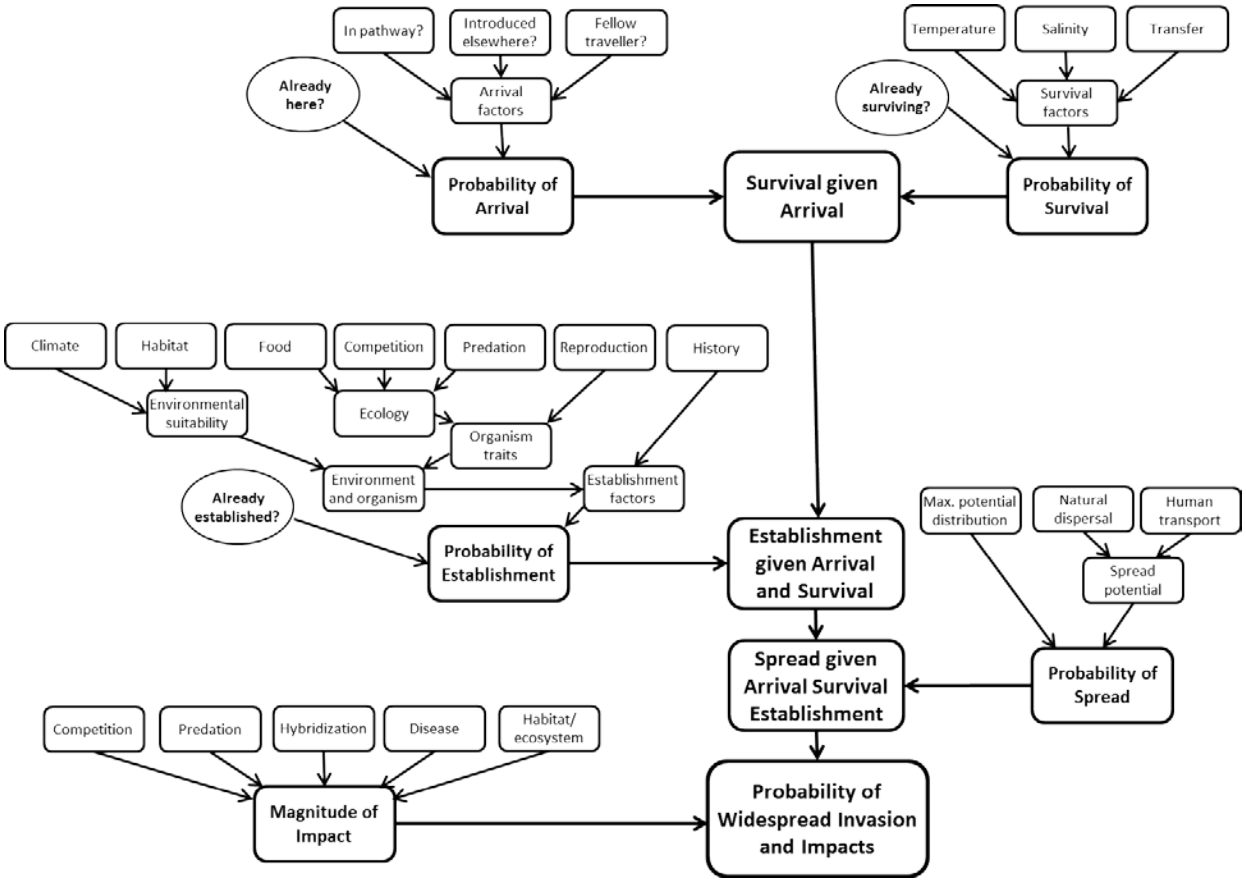


Figure 1. Schematic of the full conceptual model to assess the risk of widespread invasion and impacts of non-native aquatic species in Ontario.

Constructing the Bayesian network

The conceptual model depicting causal relationships among factors and between invasion stages was converted to a Bayesian network by: 1) defining the conditional relationships among factors and stages, and 2) incorporating uncertainty within the model.

Defining conditional relationships within the model

A Bayesian network (BN) is a probabilistic graphical model that represents a set of random variables and their conditional dependencies or relationships. These relationships are depicted using a directed graph comprised of nodes representing the variables of interest and one-way arrows or arcs that indicate the conditional dependencies between linked variables. For example, an arc from node A to node B indicates that there is a direct causal influence of A on B. Nodes that influence other nodes are parent nodes while nodes influenced by other nodes are child nodes. Nodes without parents are called root nodes.

In the Bayesian network developed (Figure 2), the answers (denoted with S) and uncertainty levels (denoted with U) selected for each of the questions in the corresponding risk assessment questionnaire (i.e., 1.01 – 5.05S/U) represent the root nodes in the network. These in turn, are the parent nodes that define the probability distributions for the score ratings. Most nodes in the network act as both parent and child nodes influencing and being influenced by other factors. Probability of widespread invasion and impacts is strictly a child node.

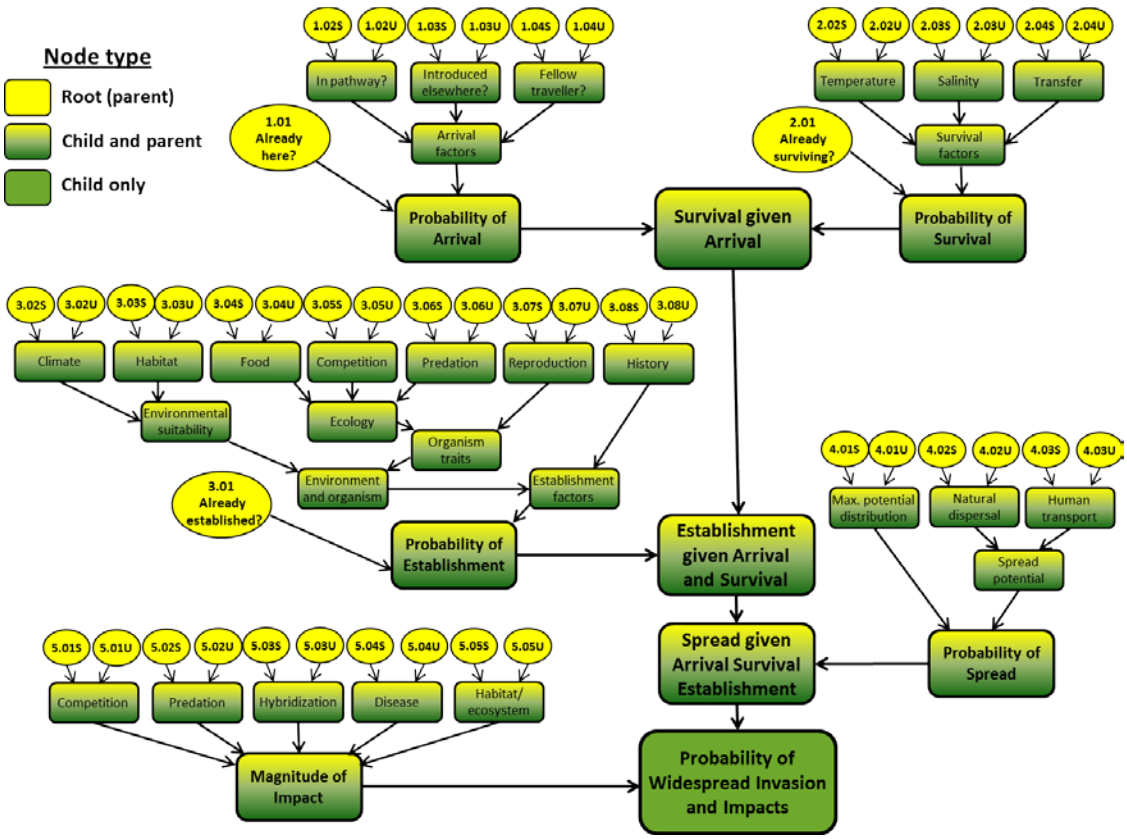


Figure 2. Schematic of the full Bayesian network with all variables in the model depicted as nodes, and all nodes categorized by type according to whether they influence and/or are influenced by other nodes.

In a Bayesian network, each node is defined by a set of discrete (i.e., not continuous) states that represent possible conditions or values for the variable of interest (see Appendix 2 for categorical ranking scales used). These possible conditions are defined by an associated probability table called a conditional probability table (CPT). CPTs "represent our belief about the probability of a node being in a state given information in the contributing nodes" which in turn may be based on "published and unpublished [empirical] data, output from analytical models, expert opinion, and personal experience" (Peterson et al. 2008).The CPT of any child node must be defined and represents its probability distribution given every possible state of its parent nodes. Nodes without parents (i.e., root nodes) have unconditional probability tables which are simply prior probability distributions for those factors.

CPTs for nodes with multiple parents (each with numerous possible states) can rapidly become very large and unfeasible to define manually. Here, risk matrices were used to provide a practical solution to the complexity that would result when completing CPTs. The idea is that the relationships between variables be represented as far as possible by

simple concepts (e.g., maximum, minimum, average, etc.) with elaboration or modification where logic demands (Schrader et al. 2012). This concept underlies the rule-based matrix model used in the EPPO PRA scheme (Holt et al. 2014), and the European Food Safety Authority Prima phacie project (MacLeod et al. 2012) to model pest risk.

Here, several different types of risk matrices or utility functions were used (Figure 3) to characterize the relationships among nodes and provide a basis for the CPTs. Descriptions for the different types of matrices indicated in Figure 3 are subsequently provided. The matrices are described for ratings of very low (VL) to very high (VH), but the same combination rules apply for all (analogous) 5-point categorical scales used in the questionnaire. These matrices do not fully define the CPTs used in the model, as explained further in the section on incorporation of uncertainty.

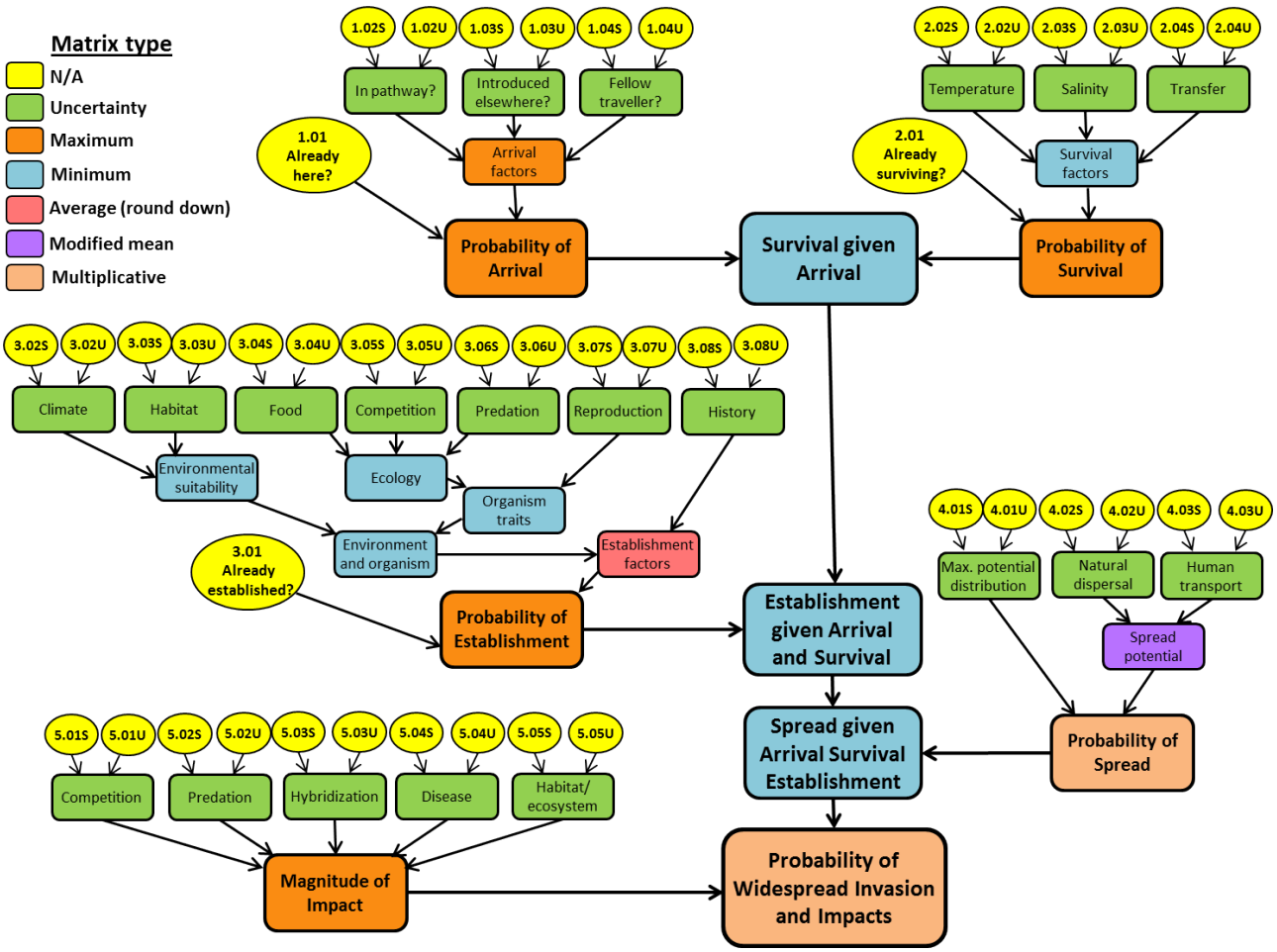


Figure 3: Bayesian network showing the different types of risk matrices used to characterize the relationships among nodes, and to provide a basis for the node conditional probability tables.

Minimum matrix

According to a minimum matrix the combination of two or more criteria or ratings is defined as the lowest of ratings such that the lowest value imposes a complete constraint over the higher value(s). “This expresses the idea of a necessary condition so that both [or all] criteria must achieve a particular rating in order for the outcome to reach that rating” (Holt et al. 2014).

Minimum matrix

	VL	L	M	H	VH
VL					
L					
M					
H					
VH					

The minimum matrix is used to define the conditional probability tables for:

Probability of survival: The minimum score of questions 2.02, 2.03, and 2.04 (Figure 3). That is, the overall probability of survival will be constrained if any of the conditions required for survival (e.g., ability to survive temperatures < 5.5 °C, ability to survive in freshwater environments, and ability to survive transfer from a pathway and introduction to the natural environment, respectively) are not satisfied.

Environmental suitability: The minimum score from questions 3.02 and 3.03 (Figure 3). Whether or not the species could successfully establish in the risk assessment area will be dependent on both the climatic and the habitat suitability, hence a minimum matrix is appropriate (Holt et al. 2014). The overall environmental suitability would be rated low if one of these factors is not suitable.

Ecology: Is defined as the minimum score of questions 3.04, 3.05, and 3.06 (Figure 3). In order for a species to successfully establish, it must not only be able to find appropriate food or nutrient resources, but also be able to overcome potential pressures from competition and/or predation. If one of these conditions is not met, the overall ecological suitability for establishment will be constrained by that factor. For example, even if it is very likely that the species could establish in Ontario despite potential competition from existing species and despite the presence of natural enemies, the overall ecological requirements for establishment would be constrained if it is very unlikely that the species would find food or other organisms necessary for its survival, growth and reproduction.

Organism traits: Since both ecological suitability (Ecology node) and reproductive traits (question 3.07) are determinants of establishment, a minimum matrix is appropriate. Both

conditions are necessary for establishment success, and thus the lowest score will constrain the overall ranking for this node.

Environment and organism: Since both environmental suitability and organism traits are likely to determine establishment, a minimum matrix is considered to be appropriate. That is, all of the factors identified and assessed in questions 3.02–3.07 are considered necessary for establishment success such that the factor with the lowest score will represent a constraint on the overall likelihood of establishment.

Probability of survival given arrival: As the probability of survival is conditional upon the species actually arriving, it will be constrained by the probability of arrival if it has a lower value than survival. Similarly, even if the probability of arrival is high, if the probability of survival is low the overall probability of both stages occurring will be constrained by survival.

Probability of establishment given arrival and survival: The probability of establishment is conditional upon the species arriving and surviving. Therefore, the lowest value for any of these stages in the invasion process will impose a constraint on the overall probability of all three occurring.

Probability of spread given arrival, survival, and establishment: The probability of spread is conditional upon the species first arriving, surviving, and establishing. Therefore, the lowest value for any of these stages in the invasion process will impose a constraint on the overall probability of all four occurring.

Maximum matrix

With a maximum matrix, the combination of two or more criteria or ratings is defined as the highest of the ratings, such that the lower rating is not a constraint on the outcome. “This expresses the idea of a sufficient condition, so that if either [or any] criterion achieves a particular rating, then the outcome also reaches that rating” (Holt et al. 2014).

Maximum matrix

	VL	L	M	H	VH
VL	VL	L	M	H	VH
L	L	L	M	H	VH
M	M	M	M	H	VH
H	H	H	H	H	VH
VH	VH	VH	VH	VH	VH

The maximum matrix is used to define the conditional probability tables for:

Probability of Arrival: Will be the maximum score of questions 1.01, 1.02, 1.03 and 1.04. That is, if any of the pathways for arrival are likely, or if the species is already known to have arrived, the overall probability of arrival will reflect only the most likely case.

Magnitude of Impact: Is defined as the maximum rating of questions 5.01, 5.02, 5.03, 5.04, and 5.05. That is, regardless of the mechanism of impact (i.e., whether through competition, predation, hybridization, disease, or habitat/ecosystem alterations), the overall magnitude of impact will reflect the largest impact rating from among these. This conforms to the unified system for classification of the magnitude of impacts of alien species as proposed by Blackburn et al. (2014) wherein “the impact category to which is a species is assigned is that corresponding to the highest level of deleterious impact identified from any of the impact mechanisms”.

Average (round down) matrix

An average (round down) matrix reflects an intermediate of two criteria, weighted towards the smaller. Being a discrete model, the result is rounded down if the intermediate of the two ratings falls on the boundary between categories. For example, to combine a rating very low and high, the intermediate or average rating would fall somewhere between low and moderate. Rounding down, the resulting rating would be therefore be low. This applies to situations where “the outcome lies between the two ratings but has a tendency to be more influenced by the lower” (Holt et al. 2014). Being an intermediate between a minimum and maximum matrix this matrix is appropriate where the choice of combination rule used to aggregate criteria is less clear than a strict maximum or minimum.

Average (round down) matrix

	VL	L	M	H	VH
VL	VL	VL	L	L	M
L	VL	L	L	M	M
M	L	L	M	M	H
H	L	M	M	H	H
VH	M	M	H	H	VH

The average round down matrix is used to define the conditional probability tables for:

Establishment factors: Since both environmental suitability/organism traits and history of establishment success are likely to influence, rather than determine establishment, an average matrix is considered to be appropriate. To account for the well-documented positive correlation between a species’ history of successful introductions and

establishment success elsewhere and its likelihood of becoming established and/or invasive in a new area, the node defined as history is given direct influence over the overall probability of establishment. It is weighted equally to all other predictors of establishment success combined (as reflected in the average matrix), though the overall probability of establishment is rounded down so as not to over-inflate the influence of one over the other.

Modified mean matrix

A modified mean matrix also represents an intermediate between two categories, and was defined specifically to capture the way in which human-assisted and natural spread combine to represent the overall spread potential of a species. In this specific case it is not appropriate to constrain overall spread by the lower of the two mechanisms of spread nor would the combination of both simply reflect the maximum of the two. In certain cases both mechanisms can combine additively such that the overall spread capacity is greater than the strict average of the contributing parts. For example, where a species is expected to have a moderate rate of natural spread, as well as a moderate rate of human-assisted spread, the overall spread potential is likely to be higher than moderate as both mechanisms of spread would be acting in concert. This matrix was defined mathematically as follows:

1) Numeric values (bounded between 0 and 1) were assigned to categorical values (i.e., from categorical lowest to highest = 0.2, 0.4, 0.6, 0.8, 1.0)

2) The arithmetic mean of the combination of all values across the matrix was calculated. E.g.:

	.2	**.4**	**.6**	**.8**	**1.0**
.2	.2	.3	.4	.5	.6
.4	.3	.4	.5	.6	.7
.6	.4	.5	.6	.7	.8
.8	.5	.6	.7	.8	.9
1.0	.6	.7	.8	.9	1.0

3) The following (conservative) rule set was applied to convert numeric values back to categorical values:

Let values: 0≤VL ≤ 0.2; 0.2<L<0.4; 0.4≤M<0.6; 0.6≤H<0.8; 0.8≤VH≤1.0

Modified mean matrix

	VL	L	M	H	VH
VL	VL	L	M	M	H
L	L	M	M	H	H
M	M	M	H	H	VH
H	M	H	H	VH	VH
VH	H	H	VH	VH	VH

As noted above, the modified mean matrix was used to define the conditional probability table for spread potential.

Multiplicative matrix

The combination of two or more criteria or ratings is defined as their product in a multiplicative matrix. This reflects the case where both events (or factors) are equally necessary to determine the outcome. It defines the probability of one factor and another factor co-occurring. By definition, this is calculated by multiplying the probabilities of both events together. If the probabilities of both are less than 1, the product is less than either one of them. This matrix was defined mathematically as follows:

1) Numeric values (bounded between 0 and 1) were assigned to categorical values (i.e., from categorical lowest to highest = 0.2, 0.4, 0.6, 0.8, 1.0)

2) The product of the combination of all values across the matrix was calculated. E.g.:

	.2	.4	.6	.8	1.0
.2	.04	.08	.12	.16	.20
.4	.08	.16	.24	.32	.40
.6	.12	.24	.36	.48	.60
.8	.16	.32	.48	.64	.80
1.0	.20	.40	.60	.80	1.0

3) The following (conservative) rule set was applied to convert numeric values back to categorical values:

Let values: 0≤VL ≤ 0.2; 0.2<L<0.4; 0.4≤M<0.6; 0.6≤H<0.8; 0.8≤VH≤1.0

Multiplicative matrix

	VL	L	M	H	VH
VL	VL	VL	VL	VL	L
L	VL	VL	L	L	M
M	VL	L	L	M	H
H	VL	L	M	H	VH
VH	L	M	H	VH	VH

The multiplicative matrix was used to define the conditional probability tables for:

Probability of Spread: The probability of spread is the product of maximum potential distribution (question 4.01) and spread potential. This takes into account that both factors are necessary to define the overall probability for spread, and that the product of both will be heavily influenced by low values. For example, consider the case where a species has a high capacity for spread but only a small part of the risk assessment area would provide suitable habitat for its establishment. Even if it were to quickly spread throughout that small region, the overall risk of spread (i.e., the probability of spreading throughout the entire risk assessment area) would be low.

Probability of widespread invasion and impacts: The probability of widespread invasion and impacts is the product of probability of spread given arrival, survival, and establishment and magnitude of impact. This adheres to the standard definition of risk, i.e., risk = likelihood x consequence.

Incorporating uncertainty

The value in using a Bayesian network to model risk is that incorporation of uncertainty is explicit: outcomes are expressed as probabilities. A further advantage of the BN method is the consistent propagation of uncertainty throughout the model (MacLeod et al. 2012).

Uncertainty is incorporated in the risk assessment tool in several ways. First, uncertainty for all question ratings is explicitly identified and documented by the risk assessor. Following the standard approach employed in the EPPO PRA scheme uncertainty is expressed on a 3-point qualitative scale (i.e., low, medium, high), and reflects the degree of confidence that the assessor has that the selected rating for a question is the correct one. Further reflecting the EPPO PRA (or PRATIQUE) approach, “low, medium and high uncertainty were defined as expressing 90, 50, and 35% confidence, respectively, that the

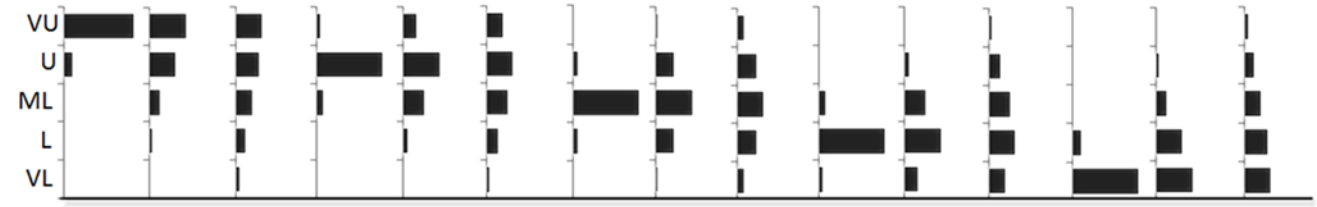
rating selected is the correct one”(Holt et al. 2011). This scale is adapted from the definitions of the Intergovernmental Panel on Climate Change relating to guidance on addressing uncertainties (IPCC 2005).

Within the BN the questionnaire ratings and uncertainty levels are translated into quantitative probability distributions. The uncertainty identified by the assessor for each question rating is captured by distributing the outcome over the range of possible ratings according to the degree of uncertainty (though the highest probability falls in the same rating category identified by the assessor). For example, if uncertainty is very low, 90% of the rating distribution would lie in the selected rating and the remaining 10% will fall across other adjacent rating categories (Holt et al. 2011).

All combinations of question ratings and uncertainty levels were assigned discrete statistical distributions (i.e., beta distribution) according to the deterministic CPT shown in Table 1 (i.e., uncertainty matrix as labelled in Figure 3). A beta distribution is suitable because it has a flexible shape and is appropriately bounded between 0 and 1 (as all probabilities by definition must be) (O’Hagan et al. 2006). The shape of the beta distribution is determined by two parameters: α and β. The mathematical derivation of these parameters for all 15 combinations of ratings and uncertainty levels are detailed in Holt et al. (2011). In the case where the assessor answers unknown to a question (in which case no associated uncertainty level is defined) the probabilities assigned to different ratings follow a uniform distribution (i.e., all ratings are assigned equal probabilities).

Table 1: The proportion of the distribution in each rating category at different levels of uncertainty, based on the beta distribution. Note: the rating categories shown here are very unlikely (VU) to very likely (VL), but the same distributions apply to all categorical ratings defined by a 5-point scale in the risk assessment questionnaire. The histogram below the table provides a pictorial representation of the same probabilities.

Selected rating		Very unlikely (VU)			Unlikely (U)			Moderately likely (ML)			Likely (L)			Very Likely (VL)		
Uncertainty		Low	Medium	High	Low	Medium	High	Low	Medium	High	Low	Medium	High	Low	Medium	High
Distributed ratings	VU	0.90	0.50	0.35	0.03	0.17	0.21	0.00	0.01	0.07	0.00	0.00	0.02	0.00	0.00	0.03
	U	0.10	0.35	0.30	0.90	0.50	0.35	0.05	0.24	0.26	0.00	0.05	0.14	0.00	0.02	0.11
	ML	0.00	0.13	0.21	0.07	0.28	0.28	0.90	0.50	0.35	0.07	0.28	0.28	0.00	0.13	0.21
	L	0.00	0.02	0.11	0.00	0.05	0.14	0.05	0.24	0.26	0.90	0.50	0.35	0.10	0.35	0.30
	VL	0.00	0.00	0.03	0.00	0.00	0.02	0.00	0.01	0.07	0.03	0.17	0.21	0.90	0.50	0.35



Uncertainty associated with the way in which two or more parent factors combine to influence a child node was also incorporated in the BN. The matrices defined above describe the general relationships between (and rules for combining) the ratings of parent nodes, but these do not include uncertainty in themselves and are therefore not technically CPTs. Consider the node for environmental suitability, which is influenced by both climate and habitat according to the rules of a minimum matrix. If the rating for climate is somewhat similar (i.e., low) and for habitat is moderately similar (i.e., medium), the combination rule states that the resultant environmental suitability would be unsuitable (i.e., low). Without incorporating uncertainty, this would give a probability of 1 to the rating unsuitable and a probability of 0 to all other ratings.

The advantage of CPTs in comparison with risk matrices is that they allow for uncertainty about the combination of ratings by assigning probability values to these combinations (Schrader et al. 2011). To incorporate uncertainty in rating combinations the outcome is distributed over the range of possible ratings for the child node with the modal (or mean) rating being the same as that identified in the corresponding risk matrix. For example, while we expect that it is most likely that the ranking for environmental suitability would be unsuitable given the parent node rankings described above we cannot be 100% certain that this will always be the case. There is a possibility that it could in fact be very unsuitable, moderately suitable, or perhaps even suitable. Substantial uncertainty always

exists when modelling complex ecological phenomena or processes and BNs offer a pragmatic means of incorporating that uncertainty within CPTs.

For all CPTs, the distribution of probabilities across rating categories follows a beta distribution with a mean identified by the risk matrix and a standard deviation of 0.1 (Note: to define a numeric mean for the probability density function, categorical ratings were given numeric values: i.e., 0.1, 0.3. 0.5, 0.7, 0.9). The parameters α and β were calculated based on these mean and standard deviation values using NtRand, an Excel Add-In Random Number Generator based on the Mersenne Twister algorithm. The general distributions used to translate the risk matrices to conditional probability tables are shown in Table 2. The full CPTs for all matrix types employed in the model are presented in Appendix 3.

Table 2. The proportion of the distribution in each rating category, according to the modal rating identified in the corresponding risk matrix for the node of interest, and based on the Beta distribution. These general distributions form the basis of all conditional probability tables for child nodes within the Bayesian network. Note: the rating categories shown here are Very low (VL) to Very high (VH), but the same distributions apply to all categorical ratings defined by a 5-point scale in the risk assessment questionnaire.

	Modal rating*	**VL**	**L**	**M**	**H**	**VH**
Parameters	**Mean/std dev**	0.1/0.1	0.3/0.1	0.5/0.1	0.7/0.1	0.9/0.1
	α/β	0.8/7.2	6/14	12/12	14/6	7.2/0.8
Distributed ratings	**VL**	0.846	0.146	0.001	0.000	0.000
	L	0.141	0.702	0.151	0.003	0.000
	M	0.013	0.149	0.689	0.149	0.013
	H	0.000	0.003	0.159	0.702	0.141
	VH	0.000	0.000	0.000	0.146	0.846

*As defined by the matrix employed to characterize the node of interest

The Bayesian network approach to risk assessment not only allows for uncertainty in the rating assignment, but also propagates this uncertainty together with uncertainties about rating combinations (Schrader et al. 2011).

Deterministic CPTs

All CPTs incorporate uncertainty except in the case where the answer to Q1.01, 2.01 or 3.01 is yes. If it is known with 100% certainty (i.e., documented evidence exists) that the species in question has already arrived (i.e., been found and positively identified in the natural environment), survived, and/or established a population in Ontario, then the CPT for probability of arrival, survival, and/or establishment reverts to a deterministic function. A yes answer overrides the answers/uncertainty distributions associated with the other factors and generates an output of very high with a probability of 1 for that invasion stage.

Software employed

A large number of BN software packages exists (Fenton and Neil 2013). Several of these were explored for building the Bayesian model defined here, including: Netica (Norsys), AgenaRisk (Agena), and GeNIe (BayesFusion).

GeNIe (Graphical Network Interface) is a development environment for building graphical decision-theoretic models based on the SMILE (structural modeling, inference, and learning engine) reasoning engine for graphical probabilistic models (i.e., GeNIe is the graphical user interface to SMILE). This software package was selected because it was available as an open source tool with full functionality, thorough documentation (including tutorials and reference manual (GeNIe_Documentation), and an online user forum to provide support. GeNIe was developed by and originally available to the community through the Decision Systems Laboratory, School of Information Sciences, University of Pittsburgh, though at the time of writing, the licence for GeNIe has now been acquired by BayesFusion, LLC.

GeNIe offers a choice of inference algorithms for belief updating in BNs including both exact and stochastic sampling algorithms. For the model developed here, a likelihood sampling algorithm is employed. The GeNIe implementation of this algorithm is based on Fung and Chang (1990).

Test cases

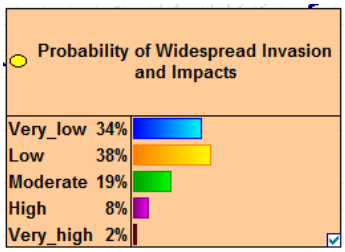
The Bayesian risk assessment tool was specifically designed to evaluate the probability of widespread invasion and impacts of non-native aquatic species in Ontario. As a result, applicability is restricted to aquatic taxa (fish, aquatic invertebrates, and aquatic plants).

The model has not been calibrated or tested for invasive aquatic parasites, viruses, or other diseases that may be incidentally introduced along with other non-native taxa.

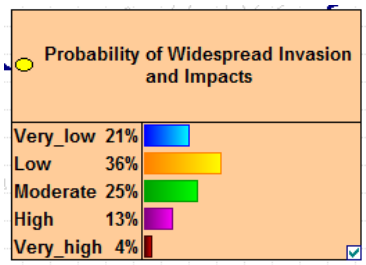
The tool was tested for five non-native aquatic species with potential pathways of arrival into Ontario: three fish species, one aquatic invertebrate, and one aquatic plant. These species represent a range of perceived risk levels for Ontario and all but one of these have been recently subject to a detailed-level, peer-reviewed risk assessment process for Ontario. Consequently, the model output can be evaluated for consistency in outcome with previously established risk assessment results.

Fish

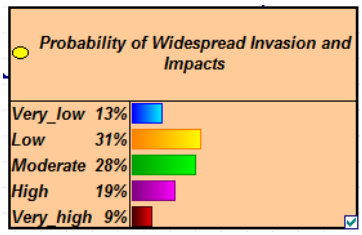
The Oriental weatherfish (*Misgurnus anguillicaudatus*) is a small freshwater species native to eastern Asia which is currently available for purchase live in Ontario through both the live food fish and the aquarium trade. In a recent, peer-reviewed detailed-level risk assessment this species was estimated to pose a low overall potential risk of invasion in Ontario (with a moderate level of uncertainty) (Nienhuis 2015). Using the Bayesian risk assessment tool to calculate the overall probability of widespread invasion and impacts for this species, the following distribution was obtained:



A peer-reviewed, detailed-level risk assessment conducted for Wels catfish (*Silurus glanis*) (a species recently identified by the Council of Great Lakes Governors as one of its least wanted aquatic invasive species) concluded that the overall risk posed by this species in Ontario was estimated to be moderate with high uncertainty (Nienhuis 2016). The output of the Bayesian risk assessment tool for this species was:

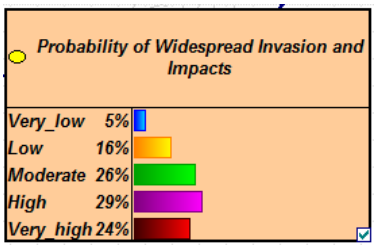


The common goldfish (*Carassius auratus*) is widely available for purchase in Ontario and has likely been subject to countless release events within natural waterbodies in the province. Despite multiple introductions there has only recently been evidence of large, established populations in the province (e.g., in Hamilton harbour). Based on the best available information used to answer the questionnaire for this species and derive model input parameters, the following probability distribution for the likelihood of widespread invasion and impacts of goldfish in Ontario was generated:



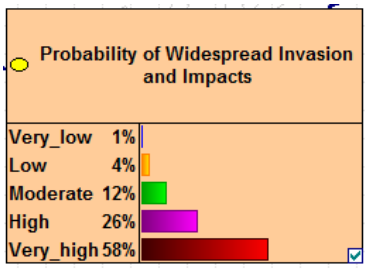
Invertebrates

The killer shrimp (*Dikerogammarus villosus*) is a large amphipod crustacean native to the Ponto-Caspian region of eastern Europe. Also on Council of Great Lakes Governor's least wanted"list, this species was identified in a peer-reviewed detailed-level risk assessment as posing a moderate overall risk of widespread invasion and impacts in Ontario (with moderate uncertainty) (Kerr 2016). The outcome of the model developed here calculated the overall probability of invasion and impacts as shown below:



Aquatic plants

Water chestnut (*Trapa natans*) has been introduced and established populations in Ontario, and was estimated in a detailed-level risk assessment to pose a high invasion risk with low uncertainty (Nienhuis 2015).The probability of widespread invasion and impacts for Ontario estimated using Bayesian inference was distributed as follows:



References

Blackburn, T.M. F. Essl, T. Evans, P.E. Hulme, J.M. Jeschke, I. Kuhn, S. Kumschick, Z. Markova, A. Mrugała, W. Nentwig, J. Pergl, P. Pysek, W. Rabitsch, A. Ricciardi, D.M. Richardson, A. Sendek, M. Vila, J.R.U. Wilson, M. Winter, P. Genovesi and S. Bacher. 2014. A unified classification of alien species based on the magnitude of their environmental impacts. PLoS Biology 12(5):e1001850.

Fenton, N. and M. Neil. 2013. Risk Assessment and Decision Analysis with Bayesian Networks. CRC Press, Taylor & Francis Group, Boca Raton, FL. 503 p.

Fung, R. and K.C. Chang. 1990.Weighting and integrating evidence for stochastic simulation in Bayesian networks. *In* Henrion, M., R. Shachter, L. Kanal and J. Lemmer (eds.). Uncertainty in Artificial Intelligence 5. Elsevier, Amsterdam.

Hayes, K.R. and S.C. Barry. 2008. Are there any consistent predictors of invasion success? Biological Invasions 10: 483-506.

Holt, J. A. Leach, J. Mumford, J. Knight and A. MacLeod. 2011. Enhancements of pest risk analysis techniques. Annex 1. PRA risk rating and uncertainty visualiser, matrix models, and invasive risk impact simulator. PRATIQUE: EU Framework 7 Research Project, Enhancements of Pest Risk Analysis Techniques (Grant Agreement No. 212459). 54 p.

Holt, J., A.W. Leach, G. Schrader, F. Petter, A. MacLeod, D.J. van der Gaag, R.H.A. Baker and J.D. Mumford. 2014. Eliciting and combining decision criteria using a limited palette of utility functions and uncertainty distributions: Illustrated by application to pest risk analysis. Risk Analysis 3: 4–16.

IPCC, 2005. Guidance notes for lead authors of the IPCC Fourth Assessment Report on addressing uncertainties. Intergovernmental Panel on Climate Change.

Kerr, S. J. 2016. A risk assessment for Killer Shrimp (Dikerogammarus villosus). Report prepared for the Natural Heritage Section, Natural Resources Conservation Policy Branch Ontario Ministry of Natural Resources and Forestry. Peterborough, Ontario.

Kolar, C.S. and D.M. Lodge. 2002. Ecological predictions and risk assessment for alien fishes in North America. Science 298: 1233–1236.

Kumschick, S., S. Bacher, W. Dawson, J. Heikkilä, A.Sendek, T. Pluess, T.B. Robinson and I. Kühn. 2012. A conceptual framework for prioritization of invasive alien species for management according to their impact. NeoBiota 15: 69–100.

Kumschick, S., S. Bacher, T. Evans, Z. Markova, J. Pergl, P. Pysek, S. Vaes-Petignat, G. van der Veer, M. Vila and W. Nentwig. 2015. Comparing impacts of alien plants and animals in Europe using a standard scoring system. Journal of Applied Ecology 52: 552–561.

MacLeod, A., H. Anderson, S. Follak, D.J. Van Der Gaag, R. Potting, O. Pruvost, J. Smith, R. Steffek, I. Vloutoglou, J. Holt, O. Karadjova, H. Kehlenbeck, G. Labonne, P. Reynaud, N. Viaene, G. Anthoine, M. Holeva, B. Hostachy, Z. Ilieva, G. Karssen, V. Krumov, P. Limon, J. Meffert, B. Niere, E. Petriva, J. Peyre, E. Pfeilstetter, W. Roelofs, F. Rothlisberger, N. Sauvion, N. Schenck, G. Shrader, T. Shroeder, S. Steinmoller, L. Tjou-Tam-Sin, V. Ventsislavov, K. Verhoeven and W. Wesemael. 2012. Pest risk assessment for the European Community plant health: A comparative approach with case studies. EFSA Supporting Publications 2012: EN-319 1053 p.

Mandrak, N.E., C. Gantz, L.A. Jones, D. Marson and B. Cudmore. 2013. Evaluation of five freshwater fish screening-level risk assessment protocols and application to non-indigenous organisms in trade in Canada. DFO Canadian Science Advisory Secretariat Research Document 2013/122. v + 125 p.

Nentwig, W., E. Kuhnel and S. Bacher. 2009. A generic impact-scoring system applied to alien mammals in Europe. Conservation Biology 24: 302–311.

Nienhuis, S. 2015. A risk assessment for European Water Chestnut (*Trapa natans*) in Ontario. Report prepared for the Natural Heritage Section, Natural Resources Conservation Policy Branch, Ontario Ministry of Natural Resources and Forestry. Peterborough, Ontario.

Nienhuis, S. 2016. A Risk Assessment for Wels Catfish (*Silurus glanis*) in Ontario. Report for Natural Heritage Section, Natural Resources Conservation Policy Branch, Ontario Ministry of Natural Resources and Forestry. Peterborough, Ontario.

Nienhuis, S. and T. Haxton 2016. A Bayesian risk assessment tool to quantify the probability of aquatic invasions in Ontario – User manual. Ontario Ministry of Natural Resources and Forestry, Science and Research Branch, Peterborough, ON. Science and Research Technical Manual TM-02.

O' Hagan, A. C.E. Buck, A. Daneshkhah, J.R. Eiser, P.H. Garthwaite, D.J. Jenkinson, J.E. Oakley and T. Rakow. 2006. Uncertain Judgements: Eliciting Expert Probabilities. John Wiley and Sons, Chichester, UK. 338 p.

Peterson, D.P., B.E. Rieman, J.B. Dunham, K.D. Fausch and M.K. Young. 2008. Analysis of trade-offs between threats of invasion by non-native brook trout (*Salvelinus fontinalis*) and intentional isolation for native westslope cutthroat trout (*Oncorhynchus clarkia lewisi*). Canadian Journal of Fisheries and Aquatic Sciences 65: 557–573.

Rixon, C.A.M., I.C. Duggan, N.M.N. Bergeron, A. Ricciardi and H. J. MacIsaac. 2005. Invasion risks posed by the aquarium trade and live fish markets on the Laurentian Great Lakes. Biodiversity and Conservation 14:1365–1381.

Schrader, G., R. Baker, D. Griessinger, A. Hart, W. Hennen, J. Holt, J. Knight, A. Leach, A. MacLeod, D. Makowski, J. Mumford, F. Petter and W. Roelofs. 2011. Best practice for quantifying uncertainty and summarising and communicating risk. EU Framework 7 Research Project, Enhancements of Pest Risk Analysis Techniques, Grant Agreement No. 212459, 36 p.

Schrader, G., A. MacLeod, F. Petter, R.H.A. Baker, S. Brunel, J. Holt, A.W. Leach and J.D. Mumford. 2012. Consistency in pest risk analysis – how can it be achieved and what are the benefits? Bulletin OEPP/EPPO Bulletin 4:3–12.

van der Veer, G. and W. Nentwig. 2015. Environmental and economic impact assessment of alien and invasive fish species in Europe using the generic impact scoring system. Ecology of Freshwater Fish, 24:646–656.

Appendix 1. List of additional literature and risk assessment schemes consulted to identify relevant factors/predictors to be incorporated into the current risk assessment model, and to build the Bayesian network.

Literature consulted

Arrival (pathways/vectors)

Copp, G.H., L. Vilizzi and R.E. Gozlan. 2010. The demography of introduction pathways, propagule pressure and occurrences of non-native freshwater fish in England. Aquatic Conservation: Marine and Freshwater Ecosystems 20(5): 595–601.

Douma, J.C., C. Robinet, L. Hemerik, M.M. Mourits, A. Roques and W. van der Werf. 2015. Development of probabilistic models for quantitative pathway analysis of plant pests introduction for the EU territory. EFSA Supporting Publication 2015:EN 809. 435 p.

Drake, J.M. and D.M. Lodge. 2004. Global hot spots of biological invasions: evaluating options for ballast-water management. Proceedings of the Royal Society B: Biological Sciences 271(1539): 575–580.

Gozlan, R.E., J.R. Britton, I. Cowx and G.H. Copp. 2010. Current knowledge on non-native freshwater fish introductions. Journal of Fish Biology 76(4): 751–786.

Hulme, P.E. 2009. Trade, transport and trouble: Managing invasive species pathways in an era of globalization. Journal of Applied Ecology 46(1): 10–18.

Jerde, C.L and M.A. Lewis. 2007. Waiting for invasions : A framework for the arrival of nonindigenous species. The American Naturalist 170: 1–9.

Jeschke, J.M. and D.L. Strayer. 2005. Invasion success of vertebrates in Europe and North America. Proceedings of the National Academy of Sciences 102(20): 7198–7202.

Keller, R.P. and D.M. Lodge. 2007. Species invasions from commerce in live aquatic organisms: Problems and possible solutions. BioScience 57(5): 428.

Marson, D., B. Cudmore, D.A.R. Drake and N.E. Mandrak. 2009. Summary of a survey of aquarium owners in Canada. Canadian Manuscript Report of Fisheries and Aquatic Sciences 2906. Fisheries and Oceans Canada, Burlington, ON. 31 p.

Marson, D., B. Cudmore, D.A.R. Drake and N.E. Mandrak. 2009b. Summary of a survey of water garden owners in Canada. Canadian Manuscript Report of Fisheries and Aquatic Sciences 2906. Fisheries and Oceans Canada, Burlington, ON . 28 p.

Williams, S.L., R.E. Crafton, R.E. Fontana, E.D. Grosholz, J. Pasari and C. Zabin. 2012. Aquatic invasive species vector risk assessments : A vector analysis of the aquarium and aquascape ('ornamental species') trades in California. University of California at Davis, Bodega Marine Laboratory. 87p.

Establishment/spread (invasive traits)

Alcaraz, C., A. Vila-Gispert and E. García-Berthou. 2005. Profiling invasive fish species: the importance of phylogeny and human use. Diversity and Distributions 11(4): 289–298.

Bajer, P.G., T.K. Cross, J.D. Lechelt, C.J. Chizinski, M.J. Weber and P.W. Sorensen. 2015. Across-ecoregion analysis suggests a hierarchy of ecological filters that regulate recruitment of a globally invasive fish. Diversity and Distributions 21: 500-510.

Blackburn, T.M., J.L. Lockwood and P. Cassey. 2015. The influence of numbers on invasion success. *Molecular Ecology*, 24, pp.1942-1953.

Blanchet, S., G. Grenouillet, O. Beauchard, P.A. Tedesco, F. Leprieur, H.H. Durr, F. Busson, T. Oberdorff and S. Brosse. 2010. Non-native species disrupt the worldwide patterns of freshwater fish body size: Implications for Bergmann's rule. Ecology Letters 13(4): 421–31.

Bomford, M., S.C. Barry and E. Lawrence. 2010. Predicting establishment success for introduced freshwater fishes: A role for climate matching. Biological Invasions 12(8): 2559–2571.

Bradie, J. and B. Leung. 2015. Pathway-level models to predict non-indigenous species establishment using propagule pressure, environmental tolerance and trait data. Journal of Applied Ecology 52(1): 100–109.

Britton, J.R., J. Cucherousset, G.D. Davies, M.J. Godard and G.H. Copp. 2010. Non-native fishes and climate change: Predicting species responses to warming temperatures in a temperate region. Freshwater Biology 55(5): 1130–1141.

Cuddington, K., W.J.S. Currie and M.A. Koops. 2014. Could an Asian carp population establish in the Great Lakes from a small introduction? Biological Invasions 16(4): 903–917.

Drake, J.M. 2007. Parental investment and fecundity, but not brain size, are associated with establishment success in introduced fishes. Functional Ecology 21(5): 963–968.

Drake, J.M. and D.M. Lodge. 2004. Global hot spots of biological invasions: evaluating options for ballast-water management. Proceedings of the Royal Society B: Biological Sciences 271(1539): 575–580.

García-Berthou, E. 2007. The characteristics of invasive fishes: What has been learned so far? Journal of Fish Biology 71: 33–55.

Hansen, G.J.A., M.J. Vander Zanden, M.J. Blum, M.K. Clayton, E.F. Hain, J. Hauxwell, M. Izzo, M.S Kornis, P.B. McIntyre, A. Mikulyuk, E. Nilsson, J.D. Olden, M. Papes and S. Sharma. 2013. Commonly rare and rarely common: Comparing population abundance of invasive and native aquatic species. PloS ONE 8(10): e77415.

Hayes, K.R. and S.C. Barry. 2008. Are there any consistent predictors of invasion success? Biological Invasions 10(4): 483–506.

Hayes, K.R. and C. Sliwa. 2003. Identifying potential marine pests--a deductive approach applied to Australia. Marine Pollution Bulletin 46(1): 91–98.

Herborg, L.-M., N.E. Mandrak, B.C. Cudmore and H.J. MacIsaac. 2007. Comparative distribution and invasion risk of snakehead (Channidae) and Asian carp (Cyprinidae) species in North America. Canadian Journal of Fisheries and Aquatic Sciences 64(12): 1723–1735.

Jeschke, J.M.. 2014. General hypotheses in invasion ecology. Diversity and Distributions 20: 1229–1234.

Jeschke, J.M. and D.L. Strayer. 2006. Determinants of vertebrate invasion success in Europe and North America. Global Change Biology 12(9): 1608–1619.

Karatayev, A.Y., L.E. Burlakova, D.K. Padilla, S.E. Mastitsky and S. Olenin. 2009. Invaders are not a random selection of species. Biological Invasions 11: 2009–2019.

Keller, R.P., J.M. Drake and D.M. Lodge. 2007. Fecundity as a basis for risk assessment of nonindigenous freshwater molluscs. Conservation Biology 21:191–200.

Keller, R.P., J. Geist, J.M. Jeschke and I. Kühn. 2011. Invasive species in Europe: Ecology, status, and policy. Environmental Sciences Europe 23:23.

Keller, R.P., D. Kocev and S. Džeroski. 2011. Trait-based risk assessment for invasive species: High performance across diverse taxonomic groups, geographic ranges and machine learning/statistical tools. Diversity and Distributions 17: 451–461.

Kolar, C.S. and D.M. Lodge. 2001. Progress in invasion biology: Predicting invaders. Trends in Ecology and Evolution 16(4): 199–204.

Kolar, C.S. and D.M. Lodge. 2002. Ecological predictions and risk assessment for alien fishes in North America. Science 298: 1233–1235.

Lockwood, J.L., P. Cassey and T.M. Blackburn. 2009. The more you introduce the more you get: The role of colonization pressure and propagule pressure in invasion ecology. Diversity and Distributions 15(5): 904–910.

Marchetti, M.P., P.B. Moyle and R. Levine. 2004. Invasive species profiling? Exploring the characteristics of non-native species across invasion stages in California. Freshwater Biology 49: 646–661.

Moyle, P.B. and M.P. Marchetti. 2006. Predicting invasion success: Freshwater fishes in California as a model. BioScience 56(6): 515–524.

Pettitt-Wade, H., K.W. Wellband, D.D. Heath and A.T. Fisk. 2015. Niche plasticity in invasive fishes in the Great Lakes. Biological Invasions 17:2565–2580.

Robinet, C., H. Kehlenbeck, D.J. Kriticos, R.H.A. Baker, A. Battisti, S. Brunel, M. Dupin, D. Eyre, M. Faccoli, Z. Ilieva, M. Kenis, J. Knight, P. Reynaud, A. Yart, W. van der Werf. 2012. A suite of models to support the quantitative assessment of spread in pest risk analysis. PloS ONE 7(10): e43366.

Romanuk, T.N., Y. Zhou, U. Brose, E.L. Berlow, R.J. Williams and N.D. Martinez. 2009. Predicting invasion success in complex ecological networks. Philosophical Transactions of the Royal Society B: Biological Sciences 364: 1743–1754.

Rosecchi, E., F. Thomas and A.J. Crivelli. 2001. Can life-history traits predict the fate of introduced species? A case study on two cyprinid fish in southern France. Freshwater Biology 46: 846–853.

Stapley, J., A.W. Santure, A.W. and S.R. Dennis. 2015. Transposable elements as agents of rapid adaptation may explain the genetic paradox of invasive species. Molecular Ecology 24:2241–2252.

Van Kleunen, M., W. Dawson, D. Schlaepfer, J.M. Jeschke and M. Fischer. 2010. Are invaders different? A conceptual framework of comparative approaches for assessing determinants of invasiveness. Ecology Letters 13: 947–958.

Vélez-Espino, L.A., M.A. Koops and S. Balshine. 2010. Invasion dynamics of round goby (*Neogobius melanostomus*) in Hamilton Harbour, Lake Ontario. Biological Invasions 12: 3861–3875.

Velez-Espino, L.A. and M.A. Koops. 2012. Capacity for increase, compensatory reserves, and catastrophes as determinants of minimum viable population in freshwater fishes. Ecological Modelling 247: 319–326.

Vila-Gispert, A., C. Alcaraz, C. and E. García-Berthou. 2005. Life-history traits of invasive fish in small Mediterranean streams. Biological Invasions 7: 107–116.

Zenni, R.D. and M.A. Nuñez. 2013. The elephant in the room: The role of failed invasions in understanding invasion biology. Oikos 122: 801–815.

Impacts

Almeida, D., A. Almodóvar, G.G. Nicola, B. Elvira and G.D. Grossman. 2012. Trophic plasticity of invasive juvenile largemouth bass *Micropterus salmoides* in Iberian streams. Fisheries Research 113: 153–158.

Almeida, D. and G.D. Grossman. 2012. Utility of direct observational methods for assessing competitive interactions between non-native and native freshwater fishes. Fisheries Management and Ecology 19: 157–166.

Angermeier, P., A. Wheeler and A. Rosenberger. 2004. Assessing the consequences of nonnative trout in headwater ecosystems in western North America. Fisheries 29: 37–41.

Blackburn, T.M. F. Essl, T. Evans, P.E. Hulme, J.M. Jeschke, I. Kuhn, S. Kumschick, Z. Markova, A. Mrugała, W. Nentwig, J. Pergl, P. Pysek, W. Rabitsch, A. Ricciardi,

D.M. Richardson, A. Sendek, M. Vila, J.R.U. Wilson, M. Winter, P. Genovesi and S. Bacher. 2014. A unified classification of alien species based on the magnitude of their environmental impacts. PLoS Biology 12(5):e1001850.

Caiola, N. and A. Sostoa. 2005. Possible reasons for the decline of two native toothcarps in the Iberian Peninsula: evidence of competition with the introduced Eastern mosquitofish. Journal of Applied Ichthyology 21: 358–363.

Dick, J.T.A., M.E. Alexander, J.M. Jeschke, A. Ricciardi, H.J. MacIsaac, T.B. Robinson, S. Kumschick , O.L.F. Weyl, A.M. Dunn, M.J. Hatcher, R.A. Paterson, K.D. Farnsworth and D.M. Richardson. 2014. Advancing impact prediction and hypothesis testing in invasion ecology using a comparative functional response approach. Biological Invasions 16: 735–753.

Evans, T., S. Kumschick, E. Dyer and T. Blackburn. 2014. Comparing determinants of alien bird impacts across two continents: implications for risk assessment and management. Ecology and Evolution 4(14): 2957–2967.

Gozlan, R.E., J.R. Britton, I. Cowx and G.H. Copp. 2010. Current knowledge on non-native freshwater fish introductions. Journal of Fish Biology 76: 751–786.

Jeschke, J.M., S. Bacher, T.M. Blackburn, J.T.A. Dick, F. Essl, T. Evans, M. Gaertner, P.E. Hulme, I. Kuhn, A. Mrugala, J. Pergl, P. Pysek, W. Rabitsch, A. Ricciardi, D.M. Richardson, A. Sendek, M. Vila, M. Winter and S. Kumschick. 2014. Defining the impact of non-native species. Conservation Biology 28: 1188–1194.

Kapuscinski, K.L., J.M. Farrell and M.A. Wilkinson. 2015. Abundance, biomass, and macrophyte consumption by rudd in Buffalo Harbor and the Niagara River, and potential herbivory by grass carp. Journal of Great Lakes Research 41: 387–395.

Kumschick, S., S. Bacher, T. Evans, Z. Markova, J. Pergl, P. Pysek, S. Vaes-Petignat, G. van der Veer, M. Vila and W.Nentwig. 2015. Comparing impacts of alien plants and animals in Europe using a standard scoring system. Journal of Applied Ecology 52: 552-561.

Kumschick, S., M. Gaertner, M. Vila, F. Essl, J.M. Jeschke, P. Pysek, A. Ricciardi, S. Bacher, T.M. Blackburn, J.T.A. Dick, T. Evans, P.E. Hulme, I. Kuhn, A. Mrugala, J. Pergl, W. Rabitsch, D.M. Richardson, A. Sendek and M. Winter. 2014. Ecological impacts of alien species: Quantification, scope, caveats, and recommendations. BioScience 65: 55–63.

Kumschick, S. and W. Nentwig. 2010. Some alien birds have as severe an impact as the most effectual alien mammals in Europe. Biological Conservation 143: 2757–2762.

Nentwig, W., E. Kuhnel and S. Bacher. 2010. A generic impact-Scoring system applied to alien mammals in Europe. Conservation Biology 24: 302–311.

Ricciardi, A., M.F. Hoopes, M.P. Marchetti and J.L. Lockwood. 2013. Progress toward understanding the ecological impacts of nonnative species. Ecological Monographs 83(3): 263–282.

Sandvik, H., B. Sæther and T. Holmern. 2013. Generic ecological impact assessments of alien species in Norway : A semi-quantitative set of criteria. Biodiversity and Conservation 22: 37-62.

Simon, K.S. and C.R. Townsend. 2003. Impacts of freshwater invaders at different levels of ecological organisation, with emphasis on salmonids and ecosystem consequences. Freshwater Biology 48: 982–994.

Thiele, J., J. Kollmann, B. Markussen and A. Otte. 2010. Impact assessment revisited: Improving the theoretical basis for management of invasive alien species. Biological Invasions 12: 2025–2035.

van der Veer, G. and W. Nentwig. 2015. Environmental and economic impact assessment of alien and invasive fish species in Europe using the generic impact scoring system. Ecology of Freshwater Fish 24: 646–656.

Wittmann, M.E., C.L. Jerde, J.G. Howeth, S.P. Maher, A.M. Deines, J.A. Jenkins, G.W. Whitledge, S.R. Burbank, W.L. Chadderton, A.R. Mahon, J.T. Tyson, C.A. Gantz, R.P. Keller, J.M. Drake and D.M. Lodge. 2014. Grass carp in the Great Lakes region: Establishment potential, expert perceptions, and re-evaluation of experimental evidence of ecological impact. Canadian Journal of Fisheries and Aquatic Sciences 71(7): 992–999.

Bayesian networks

Bressan, G.M., V.A. Oliveira, E.R. Hruschka Jr. and M.C. Nicoletti. 2009. Using Bayesian networks with rule extraction to infer the risk of weed infestation in a corn-crop. Engineering Applications of Artificial Intelligence 22(4-5): 579–592.

Das, B. 2004. Generating conditional probabilities for Bayesian networks: Easing the knowledge acquisition problem. CoRR:.1–24.

Duspohl, M., S. Frank and P. Doll. 2012. A review of Bayesian networks as a participatory modeling approach in support of sustainable environmental management. Journal of Sustainable Development 5: 1–18.

Fenton, N.E., M. Neil and J.G. Caballero. 2007. Using ranked nodes to model qualitative judgments in bayesian networks. IEEE Transactions on Knowledge and Data Engineering 19: 1420–1432.

Frank, S.K., 2015. Expert-based Bayesian Network modeling for environmental management. Frankfurt Hydrology Paper 11. Institute of Physical Geography, Goethe University Frankfurt, Frankfurt am Main, Germany. 215 p.

Fung, R. and K.-C. Chang. 1990. Weighting and integrating evidence for stochastic simulation in Bayesian networks. Uncertainty in Artificial Intelligence 5: 209–219.

Hamilton, S.H., C.A. Pollino and A.J. Jakeman. 2015. Habitat suitability modelling of rare species using Bayesian networks: Model evaluation under limited data. Ecological Modelling 299: 64–78.

Hart, B.T. and C.A. Pollino. 2008. Increased use of Bayesian network models will improve ecological risk assessments. Human and Ecological Risk Assessment: An International Journal 14(5): 851–853.

Hayes, K.R., B. Leung, R. Thresher, J.M. Dambacher and G.R. Hosack. 2013. Meeting the challenge of quantitative risk assessment for genetic control techniques: a framework and some methods applied to the common Carp (*Cyprinus carpio*) in Australia. Biological Invasions 16: 1273–1288.

Holt, J., A.W. Leach, G. Schrader, F. Petter, A. MacLeod, D.J. van der Gaag, R.H.A. Baker and J.D. Mumford. 2014. Eliciting and combining decision criteria using a limited palette of utility functions and uncertainty distributions: Illustrated by application to pest risk analysis. Risk Analysis 34(1): 4–16.

Koiter, J.R., 2006. Visualizing inference in Bayesian networks. Delft University of Technology, Delft, Netherlands. M.Sc. Thesis. 117 p.

Neil, M., N. Fenton and D. Marquez. 2007. Using Bayesian networks and simulation for data fusion and risk analysis. NATO Science for Peace and Security Series: Information and Communication Security 13.

Peterson, D.P., B.E. Rieman, J.B. Dunham, K.D. Fausch and M.K. Young. 2008. Analysis of trade-offs between threats of invasion by nonnative brook trout

(*Salvelinus fontinalis*) and intentional isolation for native westslope cutthroat trout (*Oncorhynchus clarkii lewisi*). Canadian Journal of Fisheries and Aquatic Sciences 65: 557–573.

Pollino, C.A. and C. Henderson. 2010. Bayesian networks : A guide for their application in natural resource. Landscape Logic Technical Report 14. 48 p.

Pollino, C.A., C.R. Thomas and B.T. Hart. 2012. Introduction to models and risk assessment. Human and Ecological Risk Assessment: An International Journal 18(1): 13–15.

Pollino, C.A. and B.T. Hart. 2008. Developing Bayesian network models within a risk assessment framework. Proceedings of the International Environmental Modelling and Software Society, 2008: 372–379.

Robinet, C., H. Kehlenbeck, D.J. Kriticos, R.H.A. Baker, A. Battisti, S. Brunel, M. Dupin, D. Eyre, M. Faccoli, Z. Ilieva, M. Kenis, J. Knight, P. Reynaud, A. Yart, W. van der Werf. 2012. A suite of models to support the quantitative assessment of spread in pest risk analysis. PloS ONE 7(10): e43366.

Voortman, M. 2005. Using cases to refine Bayesian networks. Delft University of Technology, Delft, Netherlands. M.Sc. Thesis. 105 p.

Zagorecki, A.T. 2010. Local probability distributions in Bayesian networks: Knowledge elicitation and inference. University of Pittsburgh, Pittsburgh, PA. PhD. Thesis. 157 p.

Risk assessment tools/schemes consulted

Alberta Risk Assessment Tool (AB RAT)

Interdepartmental Invasive Alien Species Working Group. 2008. Alberta Invasive Alien Species Risk Assessment Tool Version 3. Background Documentation. Government of Alberta, Ministry of Agriculture and Rural Development.

The initiative to develop Alberta's Invasive Alien Species Risk Assessment Tool (RAT) was spearheaded by the Inter-departmental Invasive Alien Species Working Group (IASWG). Development of the tool took seven years and the participation of over 30 biologists from various Alberta ministries. The purpose of the RAT is to provide a systematic and quantitative decision-making system to assist in the prioritization of alien species for management based on their likelihood of establishing, spreading and

adversely affecting Alberta's economic base, social values, natural resource productivity, and biodiversity.

European and Mediterranean Plant Protection Organization (EPPO) Computer Assisted Pest Risk Analysis (CAPRA)

EPPO. 2011. European Union 7th Framework Programme project PRATIQUE (Grant Agreement No. 212459).

This computer software application was developed by the European Plant Protection Organization (EPPO) Secretariat to meet one of the primary objectives of the European Union-funded PRATIQUE project. The three-year project began in March 2008, and was undertaken by a consortium of specialists and scientists from 15 organizations within the EU and other nations. The resulting CAPRA software program aims to assist pest risk analysts in running the EPPO decision-support scheme for Pest Risk Analysis (PRA), and other decision-support schemes. The EPPO PRA scheme provides risk analysts with a comprehensive series of questions that explore all the factors that must be considered in assessing risk of invasive species.

Freshwater Fish Invasiveness Scoring Kit (FISK)

Copp, G.H., R. Garthwaite and R.E. Gozlan. 2005a. Risk identification and assessment of non-native freshwater fishes: concepts and perspectives on protocols for the UK. Cefas Science Technical Report No. 129, Cefas, Lowestoft. 32 p.

Copp, G.H., R. Garthwaite and R.E. Gozlan. 2005b. Risk identification and assessment of non-native freshwater fishes: a summary of concepts and perspectives on protocols for the UK. Journal of Applied Ichthyology 21: 371–373.

Copp, G.H., L. Vilizzi, J. Mumford, G.V. Fenwick, M.J. Godard and R.E. Gozlan. 2009. Calibration of FISK, an invasive-ness screening tool for non-native freshwater fishes. Risk Analysis 29: 457–467.

FISK is an electronic tool kit developed in 2005 by the Centre for Environment, Fisheries and Aquaculture Science (Cefas)—an agency of the United Kingdom government Department for Environment, Food and Rural Affairs (Defra)—as a screening tool to assess the potential invasiveness of freshwater fish (Copp et al. 2009). The protocol is adapted directly from the widely-used semi-quantitative weed risk assessment (WRA) approach of Pheloung et al. (1999).

Great Lakes Aquatic Nuisance Species Information System Risk Assessment Tool (GLANSIS RAT)

(Available as an appendix in Mandrak et al. 2013: Evaluation of Five Freshwater Fish Screening-Level Risk Assessment Protocols and Application to Non-Indigenous Organisms in Trade in Canada)

Sturtevant, R. and E. Rutherford. 2010. Great Lakes Aquatic Nonindigenous Species Information System. (unpublished data)

The GLANSIS RAT is a questionnaire risk assessment method developed by the Great Lakes Aquatic Nuisance Species Information System group at the National Oceanographic and Atmospheric Agency, Ann Arbor, MI. The tool provides very limited user guidance.

Harmonia$^+$

D'hondt, B., S. Vanderhoeven, S. Roelandt, F. Mayer, V. Versteirt, E. Ducheyne, G. San Martin, J.-C. Grégoire, I. Stiers, S. Quoilin and E. Branquart. 2014. *Harmonia*$^+$ and *Pandora*$^+$: risk screening tools for potentially invasive organisms. Belgian Biodiversity Platform, Brussels. 63 p.

Harmonia is a recently developed scheme for the first-line risk assessment of potentially invasive alien species. It stems from a review of the former ISEIA protocol that now incorporates all stages of invasion and different types of impacts. *Harmonia*$^+$ and *Pandora*$^+$ were created as parts of the Alien Alert project, on horizon scanning for new pests and invasive species in Belgium and neighbouring areas. The Alien Alert project was performed by a consortium of eight Belgian scientific institutions, which provided expert knowledge on different facets of biological invasion and risk analysis .

The *Harmonia*$^+$ questionnaire consists of 30 core questions, which are grouped in modules representing the different stages of invasion, some of which ask for the assessor's confidence in the answers provided. In addition, text fields are included with every core question for the assessor to clarify the answer provided and mention his/her sources used. The answers can subsequently be used to calculate indices that reflect the risks posed by that organism.

Montreal Risk Assessment Tool (Montreal RAT)

(Available as an appendix in Mandrak et al. 2013: Evaluation of Five Freshwater Fish Screening-Level Risk Assessment Protocols and Application to Non-Indigenous Organisms in Trade in Canada)

Fisheries and Oceans Canada (DFO). 2012. Proceedings of the Meeting on Screening-Level Risk Assessment Prioritization Protocol for Aquatic Non-Indigenous Species; November 22-24, 2011. DFO Canadian Science Advisory Secretariat Proceedings Series 2011/068.

A recent national Canadian Science Advisory Secretariat (CSAS) science advisory process was facilitated by Fisheries and Oceans Canada (DFO) to evaluate the applicability of risk assessment protocols for screening and prioritizing aquatic nonindigenous species. During this process, CSAS meeting participants developed their own risk assessment tool using AB RAT as a starting point with modification of the questions to be aquatic and more specifically meet DFO needs. This tool is referred to as the “Montreal” Risk Assessment Tool (Montreal RAT).

Similarly to AB RAT the Montreal RAT is a questionnaire-style risk assessment comprised of 17 questions, the answers to which are given score values that are then summed. Limited guidance for the tool has been developed, and it requires testing before it can be applied. However when recently applied to freshwater fish species in trade in Canada, and evaluated based on a validation dataset, the Montreal RAT protocol performed well based on establishment and impact analyses.

New York Invasiveness Assessment Tool

Jordan, M.J., G. Moore and T.W. Weldy. 2008. Invasiveness ranking system for non-native plants of New York. Unpublished. The Nature Conservancy, Cold Spring Harbor, NY; Brooklyn Botanic Garden, Brooklyn, NY; The Nature Conservancy, Albany, NY.

New York Invasive Species Council. 2010. A regulatory system for non-native species. Final Report. 131p.

The purpose of this tool is to quantify the biological invasiveness of non-native species. It was developed with the intent to be are objective and efficient, rely upon available information, and provide outputs that are useful within the proposed regulatory system. Much of the preliminary development of these tools was accomplished by The Nature Conservancy, working with the Long Island Invasive Species Management Area

(LIISMA) and the Brooklyn Botanic Gardens, and was based on similar assessment tools used by the State of Alaska.

The Invasiveness Ranking Form serves as the invasiveness assessment tool and considers the species' known and potential distribution within New York State; ecological impacts; biological characteristics and dispersal ability; distribution within both its native landscape and other places it has been introduced; difficulty of detection and control; and likelihood of hybridizing. The forms yield numerical scores; higher scores reflect a higher ecological risk associated with a particular invasive species. Separate Invasiveness Ranking Forms were developed for Plants; Fish & Aquatic Invertebrates; Terrestrial Invertebrates; and Terrestrial Vertebrates, though all are adaptations of the New York Plant Invasiveness Ranking Form.

United Kingdom Non-Native Organism Risk Assessment Scheme (UK RAS)/ Non-Native Species Application Based Risk Assessment (NAPRA)

CABI Bioscience (CABI), Centre for Environment, Fisheries and Aquaculture Science (CEFAS), Centre for Ecology and Hydrology (CEH), Central Science Laboratory (CSL), Imperial College London (IC) and the University of Greenwich (UoG). 2005. UK Non-Native Organism Risk Assessment Scheme User Manual. Version 3.3. Defra Contract CR0293. 82 p.

Produced by a consortium of six UK institutes/universities, the non-native species risk assessment scheme being used in the UK is an adapted form of the EPPO PRA scheme. The scheme has a modular structure and provides a framework for evaluating the potential for non-native species to enter, establish, spread and cause significant impacts in all or part of the risk assessment area. As with the EPPO RAS, the UK RAS provides a consistent scheme (including detailed instructions) based on a sequence of questions to assess and document the risk of a particular species.

The corresponding NAPRA tool has been adapted directly from the EPPO CAPRA software application, and continued technical support from EPPO is acknowledged. This computer-based tool provides the template for risk assessors commissioned by the Non-native Species Secretariat (NNSS) for Great Britain (GB NNSS 2011), and has been trialled and peer-reviewed for the Scottish. In addition, the UK RAS has also been selected as the proposed detailed risk assessment schema for Ireland and Northern Ireland.

United States Department of Agriculture, Animal and Plant Health Inspection Service, Plant Protection and Quarantine Weed Risk Assessment (USDA APHIS PPQ WRA)

United States Department of Agriculture. 2015. Guidelines for the USDA-APHIS PPQ Weed Risk Assessment Process. Version 1. 126 p.

Koop, A.L., L. Fowler, L.P. Newton and B.P. Caton. 2012. Development and validation of a weed screening tool for the United States. Biological Invasions 14: 273-294.

This is a new weed risk assessment model developed by the USDA, APHIS, PPQ to evaluate the risk potential of a plant taxon becoming weedy or invasive and to assess where it might establish in the United States.

Based on the Australian WRA, the PPQ model presents a series of questions pertaining to the plant taxon's Establishment/Spread and Impact Potential (i.e., its invasive potential). It combines the question–answer style of rapid screening tools with the structure typically associated with pest risk analysis. At the core of the PPQ WRA is a logistic regression risk model that describes the risk potential of the plant taxon under assessment. The model uses the risk scores from the Establishment/ Spread and impact risk elements to determine the likelihood that a given plant taxon will be a non-, minor-, or major-invader.

Appendix 2

All answer options within the risk assessment questionnaire take the form of discrete, categorical ratings. Most follow a 5-point scale (not including the option for unknown) which is consistent with many other risk assessment schemes (including EPPO CAPRA and UK RAS). The use of a 5-point scale is believed to "provide an appropriate balance between resolution and simplicity"*(* CABI Bioscience et al. 2005), and gives the risk assessor the latitude to make finer scale judgements when distinction between categories is clear.

A 3-point scale was used where discrimination between answer options at a higher level of resolution was more difficult (e.g., for question 4.03 pertaining to human-assisted dispersal). Questions associated with arrival and survival are answered more simply as either yes/no (or unknown). This rating scale serves to quickly screen in or out species for which further assessment is or is not warranted.

Table A.2. Categorical rating scales used in the risk assessment questionnaire.

Type		Rating scale				
Likelihood	***5-point***	Very unlikely	Unlikely	Moderately likely	Likely	Very likely
	2-point	No				Yes
Similarity	***5-point***	Not similar	Somewhat similar	Moderately similar	Largely similar	Completely similar
Extent	***5-point***	Never	Not widely	Moderately widely	Widely	Very widely
		Very limited	Limited	Moderately extensive	Extensive	Very extensive
		Very low	Low	Moderate	High	Very high
	3-point		Low	Moderate	HIgh	
Effect	***5-point***	Minimal	Minor	Moderate	Major	Massive
Probability	***5-point***	Very low	Low	Moderate	High	Very high

Appendix 3. Full conditional probability tables (CPTs) defined in the Bayesian network model for assessing the risk of widespread invasion and impacts of non-native aquatic species in Ontario.

Table A.3.1. CPT for child node with 2 parents as defined by minimum matrix

	Parent 1	vl					l					m					h					vh				
	Parent 2	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh
Child	vl	0.846	0.846	0.846	0.846	0.846	0.846	0.146	0.146	0.146	0.146	0.846	0.146	0.001	0.001	0.001	0.846	0.146	0.001	0.000	0.000	0.846	0.146	0.001	0.000	0.000
	l	0.141	0.141	0.141	0.141	0.141	0.141	0.702	0.702	0.702	0.702	0.141	0.702	0.151	0.151	0.151	0.141	0.702	0.151	0.003	0.003	0.141	0.702	0.151	0.003	0.000
	m	0.013	0.013	0.013	0.013	0.013	0.013	0.149	0.149	0.149	0.149	0.013	0.149	0.689	0.689	0.689	0.013	0.149	0.689	0.149	0.149	0.013	0.149	0.689	0.149	0.013
	h	0.000	0.000	0.000	0.000	0.000	0.000	0.003	0.003	0.003	0.003	0.000	0.003	0.159	0.159	0.159	0.000	0.003	0.159	0.702	0.702	0.000	0.003	0.159	0.702	0.141
	vh	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.146	0.146	0.000	0.000	0.000	0.146	0.846

Table A.3.2. CPT for child node with 2 parents as defined by maximum matrix

	Parent 1	vl					l					m					h					vh				
	Parent 2	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh
Child	vl	0.846	0.146	0.001	0.000	0.000	0.146	0.146	0.001	0.000	0.000	0.001	0.001	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	l	0.141	0.702	0.151	0.003	0.000	0.702	0.702	0.151	0.003	0.000	0.151	0.151	0.151	0.003	0.000	0.003	0.003	0.003	0.003	0.000	0.000	0.000	0.000	0.000	0.000
	m	0.013	0.149	0.689	0.149	0.013	0.149	0.149	0.689	0.149	0.013	0.689	0.689	0.689	0.149	0.013	0.149	0.149	0.149	0.149	0.013	0.013	0.013	0.013	0.013	0.013
	h	0.000	0.003	0.159	0.702	0.141	0.003	0.003	0.159	0.702	0.141	0.159	0.159	0.159	0.702	0.141	0.702	0.702	0.702	0.702	0.141	0.141	0.141	0.141	0.141	0.141
	vh	0.000	0.000	0.000	0.146	0.846	0.000	0.000	0.000	0.146	0.846	0.000	0.000	0.000	0.146	0.846	0.146	0.146	0.146	0.146	0.846	0.846	0.846	0.846	0.846	0.846

Table A.3.3. CPT for child node with 2 parents as defined by average (round down) matrix

	Parent 1	vl					l					m					h					vh				
	Parent 2	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh
Child	vl	0.846	0.846	0.146	0.146	0.001	0.846	0.146	0.146	0.001	0.001	0.146	0.146	0.001	0.001	0.000	0.146	0.001	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.000
	l	0.141	0.141	0.702	0.702	0.151	0.141	0.702	0.702	0.151	0.151	0.702	0.702	0.151	0.151	0.003	0.702	0.151	0.151	0.003	0.003	0.151	0.151	0.003	0.003	0.000
	m	0.013	0.013	0.149	0.149	0.689	0.013	0.149	0.149	0.689	0.689	0.149	0.149	0.689	0.689	0.149	0.149	0.689	0.689	0.149	0.149	0.689	0.689	0.149	0.149	0.013
	h	0.000	0.000	0.003	0.003	0.159	0.000	0.003	0.003	0.159	0.159	0.003	0.003	0.159	0.159	0.702	0.003	0.159	0.159	0.702	0.702	0.159	0.159	0.702	0.702	0.141
	vh	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.146	0.000	0.000	0.000	0.146	0.146	0.000	0.000	0.146	0.146	0.846

Table A.3.4. CPT for child node with 2 parents as defined by mean (precautionary) matrix

	Parent 1	vl					l					m					h					vh				
	Parent 2	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh
Child	vl	0.846	0.146	0.001	0.001	0.000	0.146	0.001	0.001	0.000	0.000	0.001	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	l	0.141	0.702	0.151	0.151	0.003	0.702	0.151	0.151	0.003	0.003	0.151	0.151	0.003	0.003	0.000	0.151	0.003	0.003	0.000	0.000	0.003	0.003	0.000	0.000	0.000
	m	0.013	0.149	0.689	0.689	0.149	0.149	0.689	0.689	0.149	0.149	0.689	0.689	0.149	0.149	0.013	0.689	0.149	0.149	0.013	0.013	0.149	0.149	0.013	0.013	0.013
	h	0.000	0.003	0.159	0.159	0.702	0.003	0.159	0.159	0.702	0.702	0.159	0.159	0.702	0.702	0.141	0.159	0.702	0.702	0.141	0.141	0.702	0.702	0.141	0.141	0.141
	vh	0.000	0.000	0.000	0.000	0.146	0.000	0.000	0.000	0.146	0.146	0.000	0.000	0.146	0.146	0.846	0.000	0.146	0.146	0.846	0.846	0.146	0.146	0.846	0.846	0.846

Table A.3.5. CPT for child node with 2 parents as defined by multiplicative matrix

	Parent 1	vl					l					m					h					vh				
	Parent 2	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh	vl	l	m	h	vh
Child	vl	0.846	0.846	0.846	0.846	0.146	0.846	0.846	0.146	0.146	0.001	0.846	0.146	0.146	0.001	0.000	0.846	0.146	0.001	0.000	0.000	0.146	0.001	0.000	0.000	0.000
	l	0.141	0.141	0.141	0.141	0.702	0.141	0.141	0.702	0.702	0.151	0.141	0.702	0.702	0.151	0.003	0.141	0.702	0.151	0.003	0.000	0.702	0.151	0.003	0.000	0.000
	m	0.013	0.013	0.013	0.013	0.149	0.013	0.013	0.149	0.149	0.689	0.013	0.149	0.149	0.689	0.149	0.013	0.149	0.689	0.149	0.013	0.149	0.689	0.149	0.013	0.013
	h	0.000	0.000	0.000	0.000	0.003	0.000	0.000	0.003	0.003	0.159	0.000	0.003	0.003	0.159	0.702	0.000	0.003	0.159	0.702	0.141	0.003	0.159	0.702	0.141	0.141
	vh	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.146	0.000	0.000	0.000	0.146	0.846	0.000	0.000	0.146	0.846	0.846

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